The New Mathematics
for Primary Teachers

1. Numbers and Number Systems

D. Paling and J. L. Fox

OXFORD UNIVERSITY PRESS

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THE NEW MATHEMATICS
FOR PRIMARY TEACHERS

1. Numbers and Number Systems

D. Paling and J. L. Fox

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Preface to the series

At the present time there is a growing dissatisfaction with mathematical work in schools. There is a feeling that the content of courses does not fit in with the present-day needs of science and technology, industry and trade. At the same time the attitude of teachers towards the teaching of mathematics is undergoing a fundamental change. More and more teachers are coming to feel that teaching by 'rule' is not good enough. They want children to find out more for themselves, to question what they are doing and to find their own ways of solving problems. They want children to understand rather than to learn like parrots.

These two movements—the modernisation of the content of mathematics and the shift of emphasis from 'the teacher teaching' to 'the child learning'—are going on together in many countries. They both make large demands on teachers, especially in the primary school. These teachers now need to take a new look at mathematics itself and at the same time try to introduce more enlightened methods in the classroom.

If these changes are to come about, teachers in schools and students in training will need all the help they can be given. It is for this purpose that this series of short books has been written. The aim is to give teachers and student-teachers a new mathematical background so that they can introduce modern mathematics in the primary school. The books do not set out to deal with the classroom situation itself but they do present, at teacher level, the kind of approach now being introduced into schools.

The first five books will provide a background to the first three years of mathematical work in the primary school. Further books will cater for the whole primary range. The approach is simple and straightforward and no previous knowledge is assumed. Many exercises (with answers) are provided at each stage; they need to be worked through very carefully. The exercises not only consolidate the work already done but provide the starting-point for the next stage.

St Luke's College, Exeter
March, 1968

D.P.
J.L.F.

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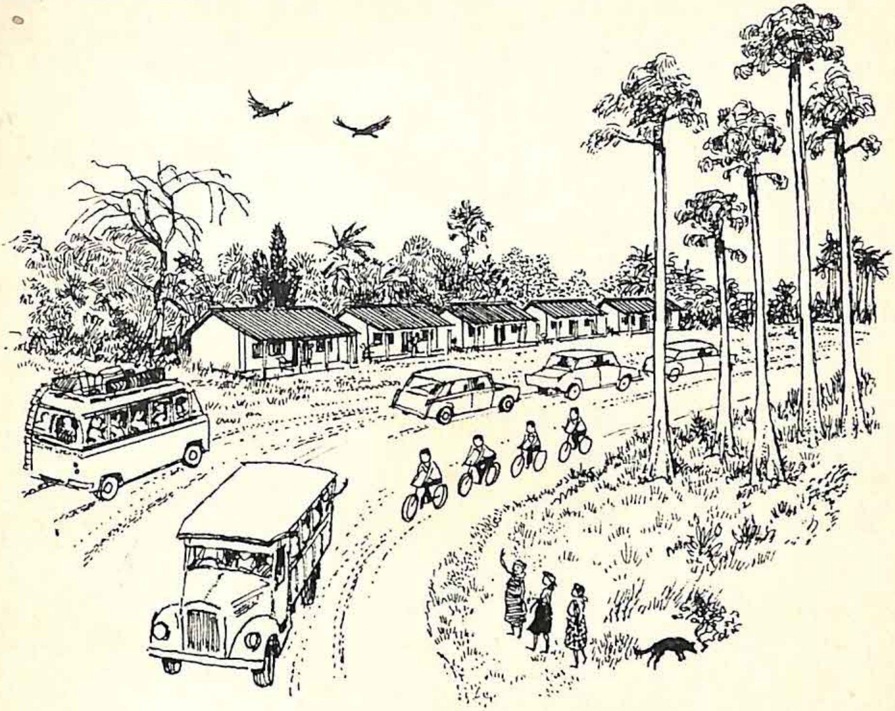
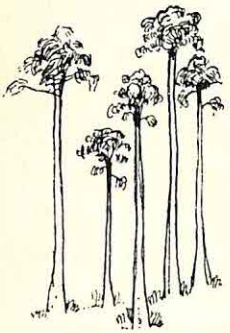
1. ONE-TO-ONE

A. A new word

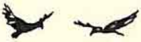


Here is a picture of a busy road? What do we see?

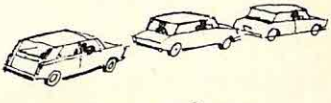


a group of trees

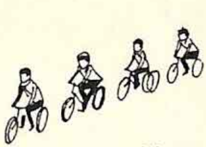
a group of women



some birds



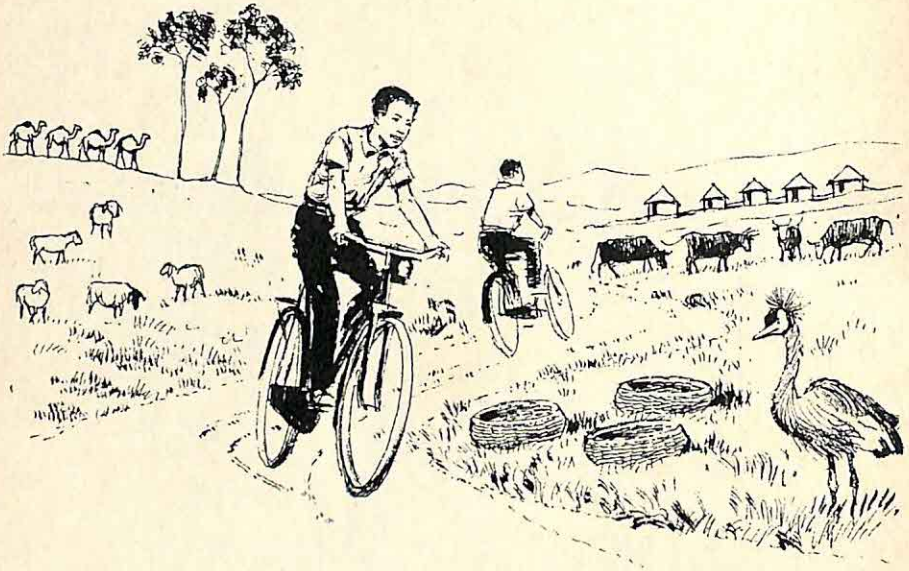
a row of cars



some cyclists

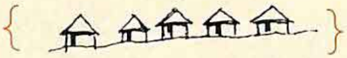
The phrases that are written under the small pictures are those we use in everyday life. In mathematics, however, we find it better to say *a set of trees*, *a set of cars*, *a set of women*, *a set of birds*, *a set of cyclists*. We use set instead of 'some', 'row', 'group'.

Many of us may wonder why we use *set* rather than 'group'. We could use 'group', but later on, this word is used for another purpose. Using the word *set* prevents confusion.

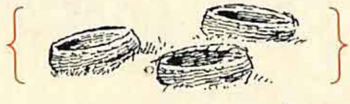


Look at the picture. What do we see?

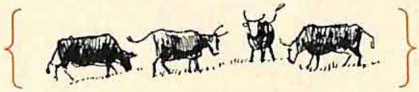
a set of huts



a set of baskets



a set of cows



To show clearly what is in each set we have enclosed its members in curly brackets.

Exercise

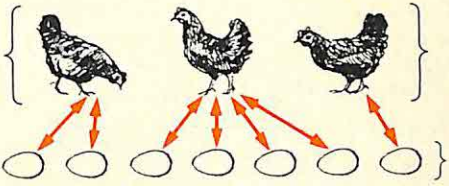
1. Sketch some of the other sets in the picture.

B. Matching

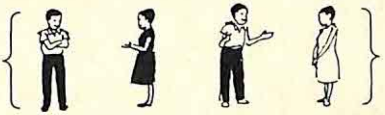
Here is a set of hens.

Here is a set of eggs which they have laid.

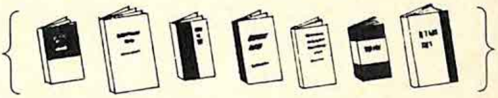
Here we show which eggs were laid by each hen.



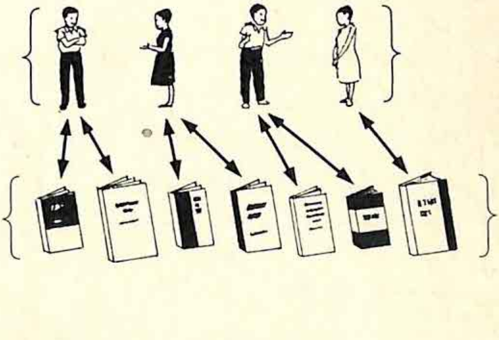
Here is a set of students.



Here is a set of books which they are taking to their lectures.



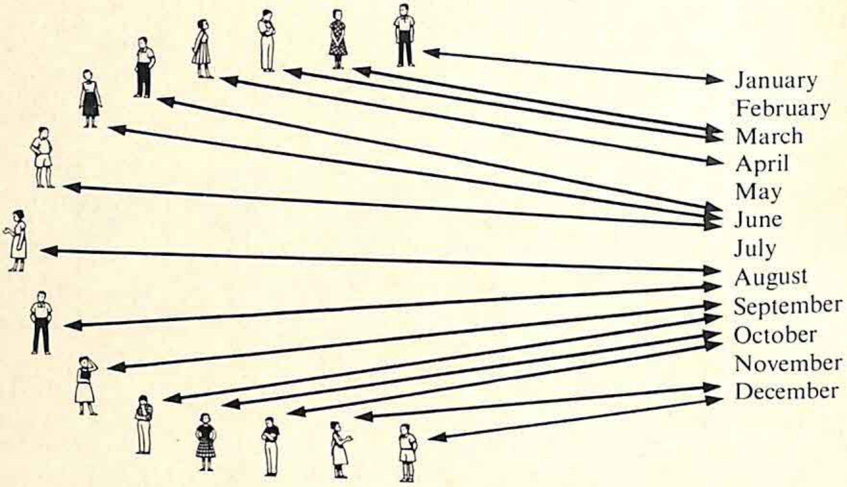
Here we show which books each student is taking.



In both of these examples we show how the members of one set are related to those of another. We draw lines between the corresponding members of the sets. We say that we match the members of one set with those of the other.

Exercises

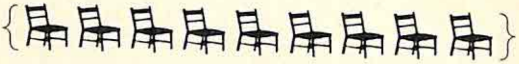
2. Here is another matching. It shows a set of people and the months in which they were born.



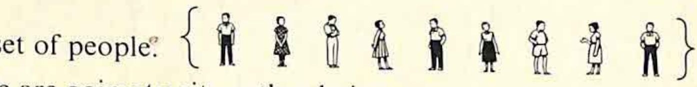
List the month of birth of several of your friends. Show this information as above.

3. List the name and the age of several of your friends. Draw a diagram to show the matching of names and ages.

4. (a) Here is a set of chairs.

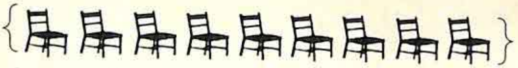


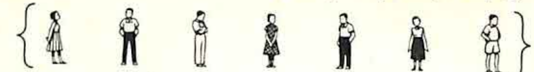
Here is a set of people.



The people are going to sit on the chairs.
Draw a diagram to show a matching between the people and the chairs.

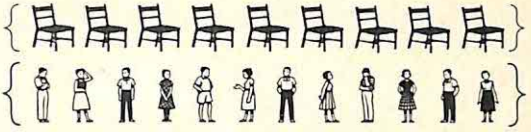
(b) Here is the same set of chairs and another set of people.





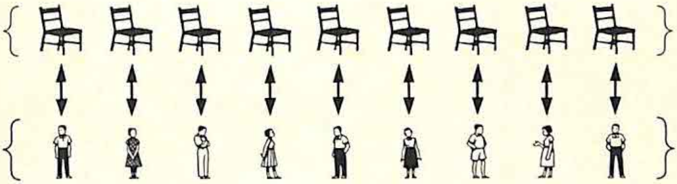
Draw a diagram to show a matching between this set of people and the set of chairs.

(c) Here again is the same set of chairs and yet another set of people. Draw a diagram to show a matching between this set of people and the set of chairs.

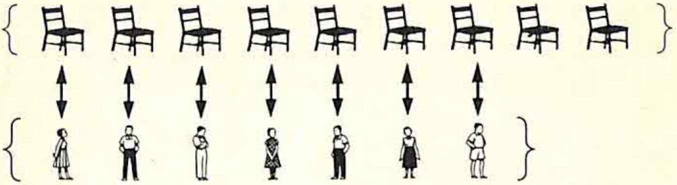


C. One-to-one correspondence

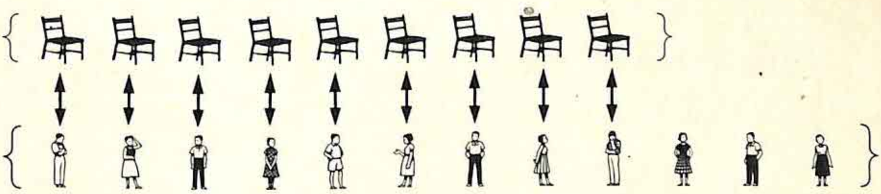
In exercise 4 we see that there are three different situations.
In (a) there is a chair for each person and a person for each chair.



In (b) there is a chair for each person but not all the chairs are occupied.

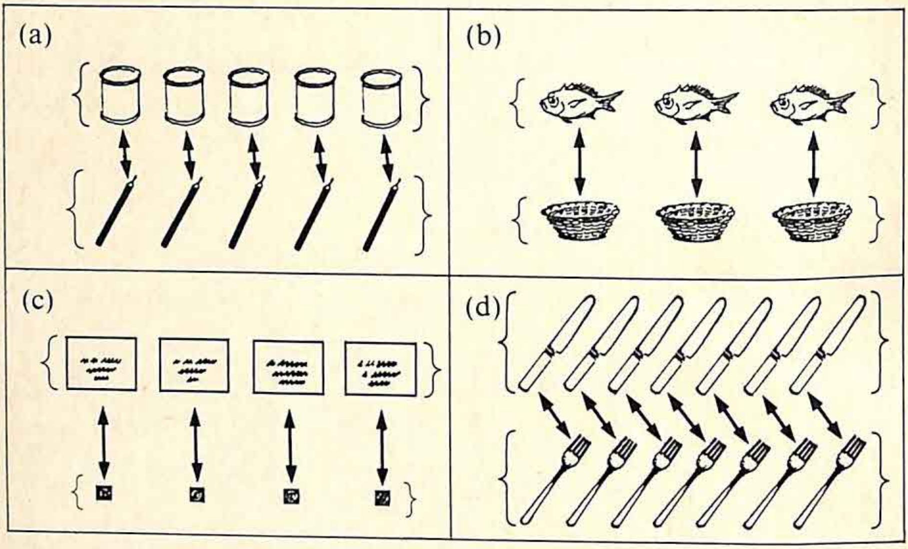


In (c) there is a person for each chair but some people have no chair on which to sit.

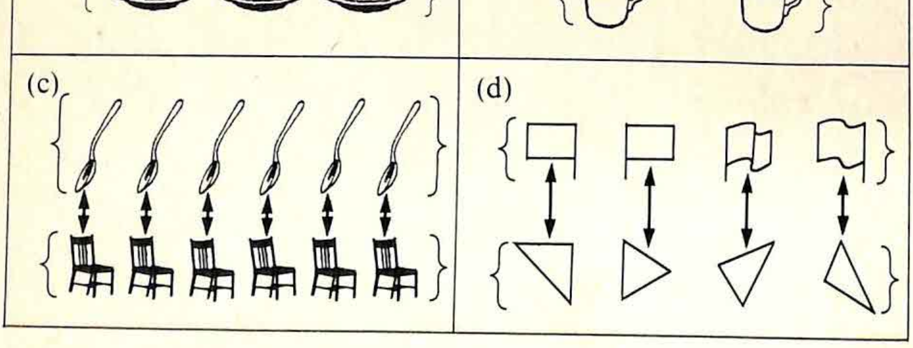


We use a special phrase to describe the situation in (a). We say that there is a one-to-one correspondence between the people and the chairs. For each person there is a chair, for each chair there is a person.

Here are some more examples of one-to-one correspondence.



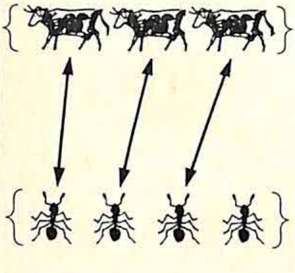
The members of the two sets in each of the above examples are related to each other in everyday life. This, however, is not true for all examples of one-to-one correspondence. Here are some examples of one-to-one correspondence where the members of the sets are not related in everyday life.



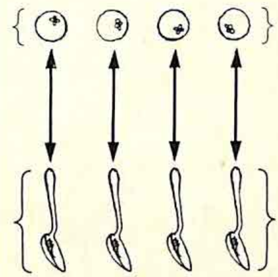
Exercises

5. Which of the following matchings are examples of one-to-one correspondence?

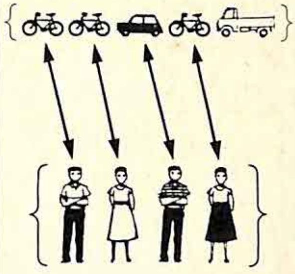
(a)



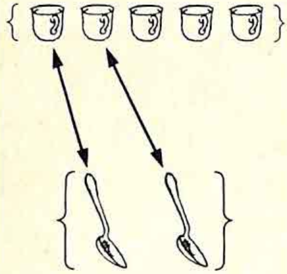
(b)



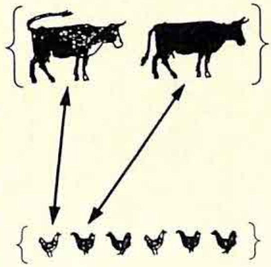
(c)



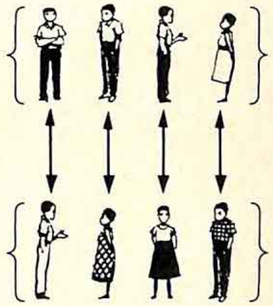
(d)



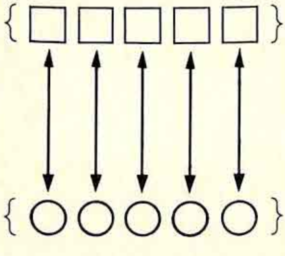
(e)



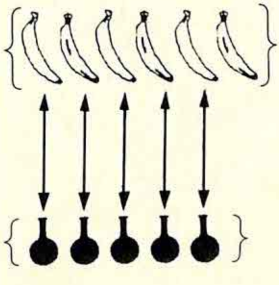
(f)



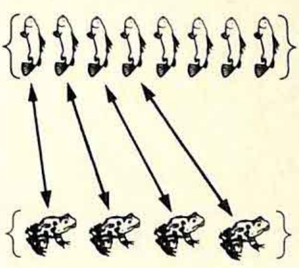
(g)



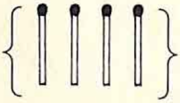
(h)

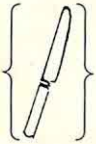


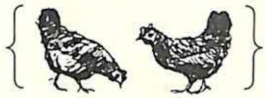
(i)

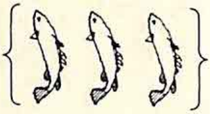


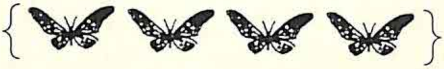
6. Here are some sets. They each have a reference letter.

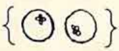
A . . .


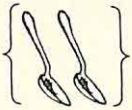
B . . .


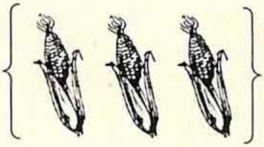
C . . .


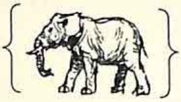
D . . .


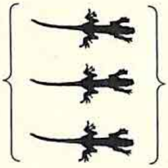
E . . .


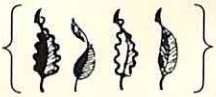
F . . .


G . . .


H . . .


I . . .


J . . .


K . . .


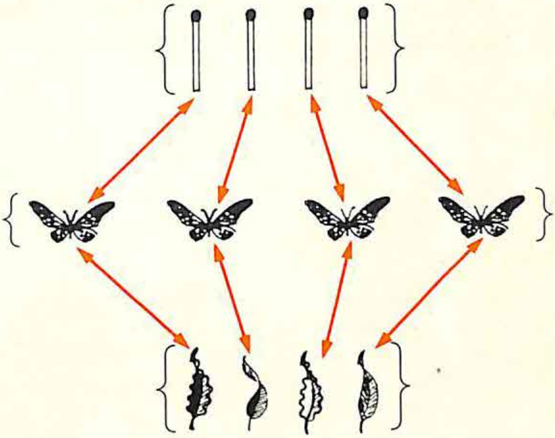
(a) Look at set *A*, the set of matches. Which of the other sets are in one-to-one correspondence with this set of matches?

(b) Which sets are in one-to-one correspondence with set *C*, the set of hens?

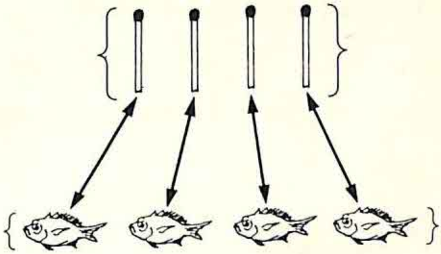
(c) Go on to deal with set *B* and then set *D* in the same way.

D. Number

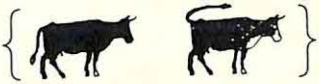
From exercise 6 we see that there is a one-to-one correspondence between sets *A*, *E*, and *K* as shown below.



We can, of course, for each of these sets, show other sets with which there is a one-to-one correspondence. For example:



At the same time we know that there are very many sets which are not in one-to-one correspondence with any of these sets. For example:



is not in one-to-one correspondence with

This makes clear the idea of a number. We say that sets which are in one-to-one correspondence have the same number. For example, every set which is in one-to-one correspondence with the set of matches has the same number. This we call four. We say that the number of the set is four. Other sets, of course, have other numbers. For example:

The number of { } is one.

The number of { } is two.

The number of { } is three.

In this way, using the idea of one-to-one correspondence, we give a number to every set of objects.

There is one more set, and its number, we need to consider. We can do this by looking at a simple example.

Jacob has some hens. They are shown here: { }. One day he takes them to market and sells them. The set of hens which he now has can be shown like this: { }. The set is empty. It is called an empty set. We say that the number of this set is nought.

Important ideas in this chapter

1. The idea of a set.
2. The idea of matching.
3. The idea of one-to-one correspondence.
4. The idea of a number.

2. RECORDING BEFORE COUNTING

A. Stones and sticks

Let us go back many years and look at some of the ways in which men kept records of their animals and other belongings.

Here is a man and his herd of goats.

It is early morning and he is taking his goats out for grazing. Later in the day when he brings the herd back, he wants to be sure that he has not lost any of them. He cannot count beyond two or three. Instead he uses small stones to keep a record. As each goat passes him he puts down a stone. When the goats have all gone he has a heap of stones. Later in the day the goats return. Now as each goat passes him he takes a stone from the heap. If he picks up the last stone as the last goat passes him he knows that all his goats are back. If he has any stones left over he knows that some of his goats are lost.

Exercise

1. What can you say about the set of stones and the set of goats at the beginning of the day?

Here is another man with his goats.

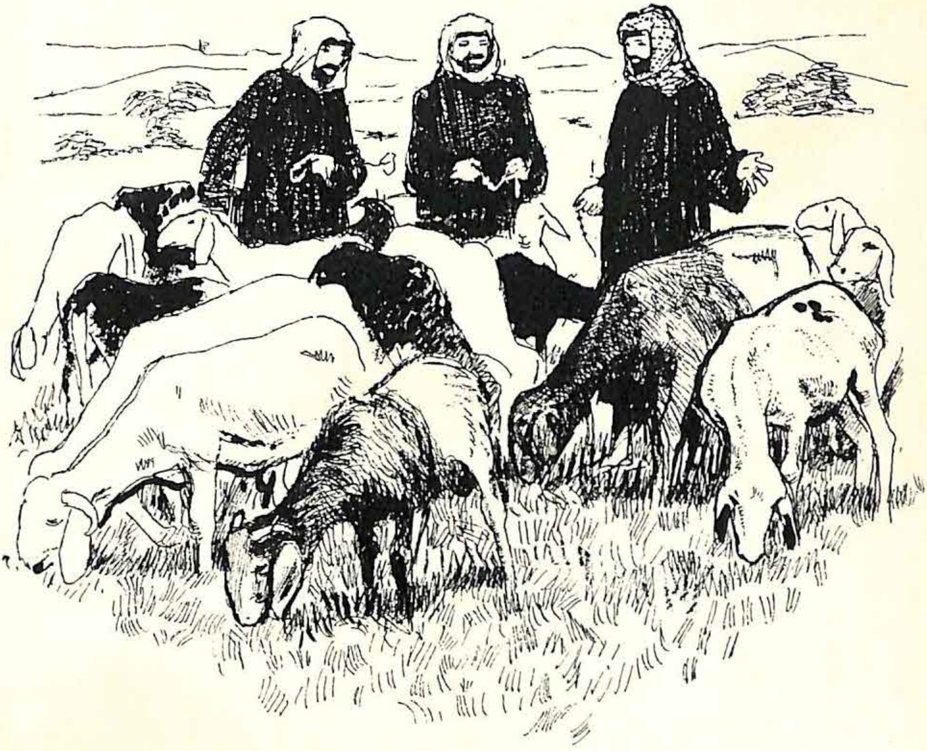
He too cannot count. However, he is not using stones either. As each goat passes him he makes a cut on a stick. When the goats come back he makes sure that there is a goat for each of the cuts.

Exercises

2. What can you say about the set of cuts and the set of goats?
3. Why is cutting a stick better than putting down stones for keeping a record?
4. Suggest other ways of keeping records.

B. Knots, cords, and bundles

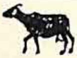
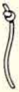
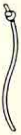
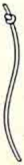
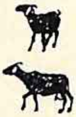
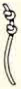
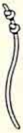
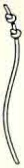
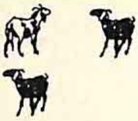
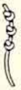
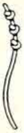
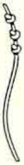
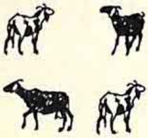
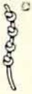
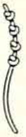
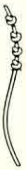
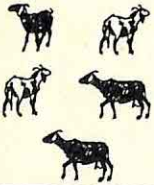
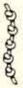
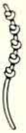
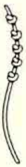
Here are three men, Tani, Jacob, and Ali. They have come from different places and are here to buy goats.



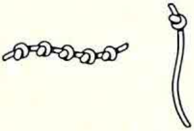
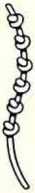
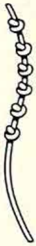
To keep a record they do not, however, use either stones or cuts. They tie knots in pieces of cord, like this:

They tie a knot for each goat.

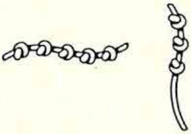
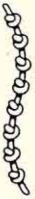
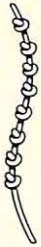
The pictures below show how each man records the goats he buys.

	Tani	Jacob	Ali
			
			
			
			
			

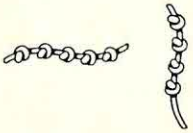
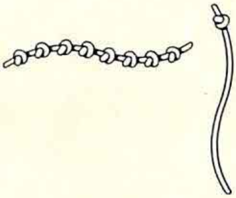
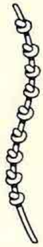
We see that the cord that Tani is using is now full. He puts it on the ground and starts another cord. Jacob and Ali do not yet need new cords. For the next goat the cords look like this:

Tani	Jacob	Ali
		

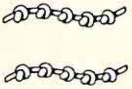
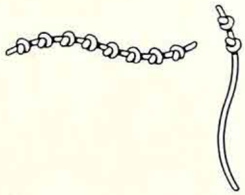
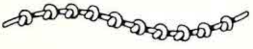
After two more goats the cords look like this:

Tani	Jacob	Ali
		

Jacob now has a full cord. He puts it on the ground and starts another for the next goat.

Tani	Jacob	Ali
		

After recording one more goat Ali has a full cord. At the same time Tani has another full cord. So their cords now look like this:

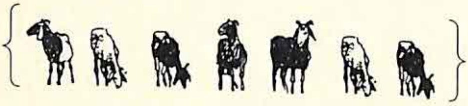
Tani	Jacob	Ali
		

They go on recording in this way.

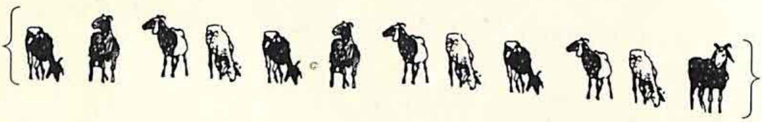
Exercises

5. Show how Tani, Jacob, and Ali record:

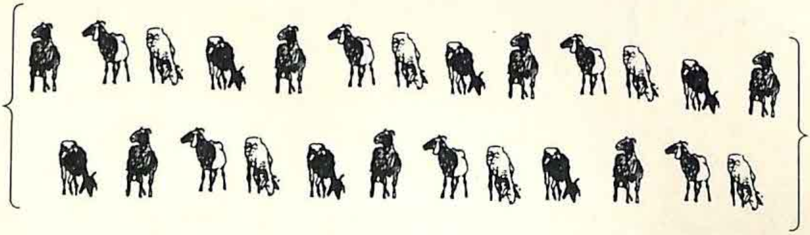
(a)



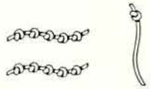
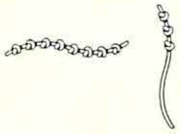
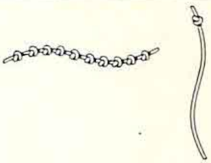
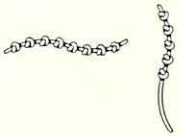
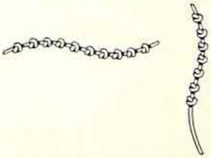
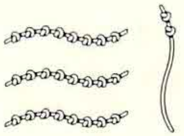
(b)



(c)

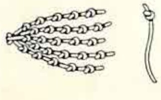
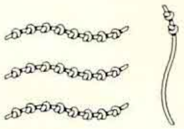
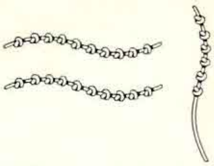


6. In (a) below we see how Tani, Jacob, and Ali each record the same number of goats. Copy the table and fill in the spaces.

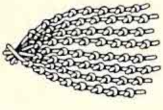
	Tani	Jacob	Ali
(a)			
(b)			
(c)			
(d)			

In the answer to exercise 5(c) we see that the number of Tani's full cords is the same as the number of knots on each of his full cords. To keep the full cords tidy he now makes a bundle of them.

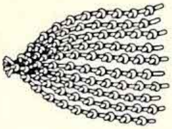
In exercise 6(d) there is one more goat to record than in exercise 5(c). Tani now has a cord with one knot as well as a bundle of cords. The answer to 6(d) is shown as:

Tani	Jacob	Ali
		

Later, when Jacob's full cords are as shown, he too makes a bundle.

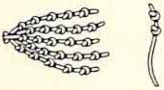
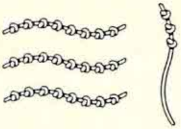
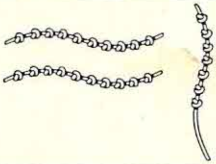
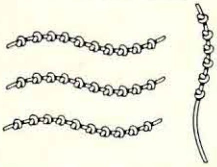
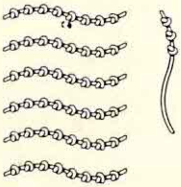
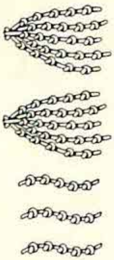


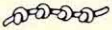
Later still, when Ali's full cords are as shown, he also makes a bundle.

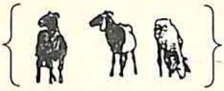


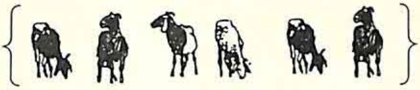
Exercises

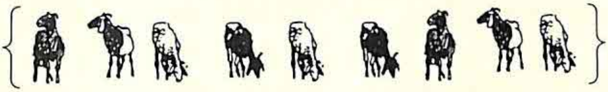
7. In (a) below we see how Tani, Jacob, and Ali each record the same number of goats. Copy the table and fill in the spaces.

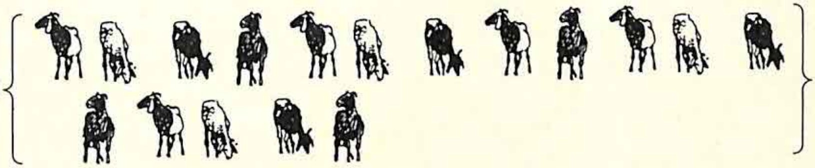
	Tani	Jacob	Ali
(a)			
(b)			
(c)			
(d)			

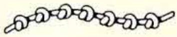
8. Here is a full cord for another man, Ahmed. 
Show how he records:

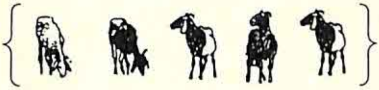
(a) 

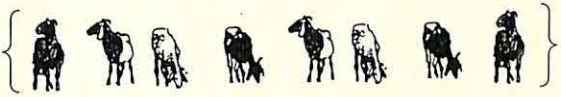
(b) 

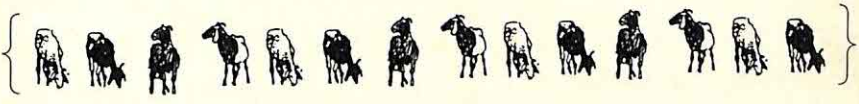
(c) 

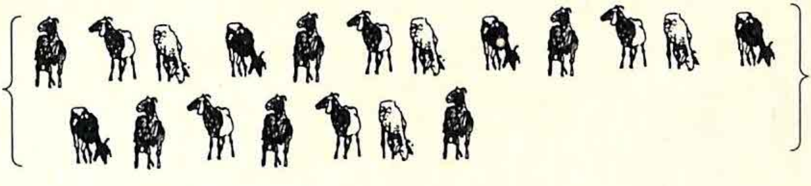
(d) 

9. Here is a full cord for another man, John. 
Show how he records:

(a) 

(b) 

(c) 

(d) 

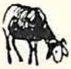
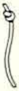
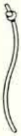
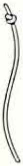
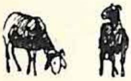
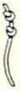
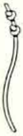
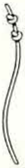
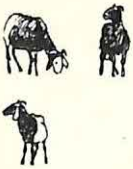
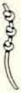
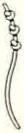
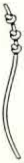
Important ideas in this chapter

1. Recording with stones, cuts in a stick, etc.
2. Recording with knots in cords.
3. The idea of knots, cords, and bundles.
4. The use of different numbers of knots on full cords.

3. SPOKEN NUMBER NAMES

In the previous chapter we saw how Tani, Jacob, and Ali kept a record of their goats. We now go on to see what happened many years later when man began to use more number names. We look at three more men—Tani, Jacob, and Ali—with their goats. (Of course, these are not the same three men as in the previous chapter. These are their descendants. They still use the same system of knots for recording.)

The number names which they say are given below.

	Tani says	Jacob says	Ali says
	 one knot	 one knot	 one knot
	 two knots	 two knots	 two knots
	 three knots	 three knots	 three knots

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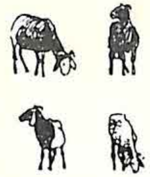
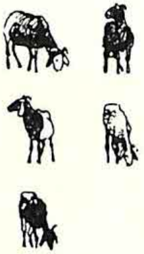
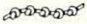
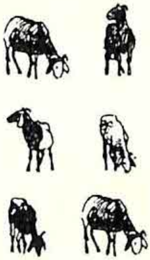
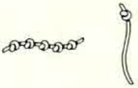
	Tani says	Jacob says	Ali says
	 four knots	 four knots	 four knots

When the next goat is recorded Tani has a full cord. He sees his full cord and says one cord. Jacob and Ali, however, need the new number name *five*.

	Tani says	Jacob says	Ali says
	 one cord	 five knots	 five knots

Now another goat is recorded. Tani says one cord, one knot, but Jacob and Ali need to use the new number name *six*.

	Tani says	Jacob says	Ali says
	 one cord, one knot	 six knots	 six knots

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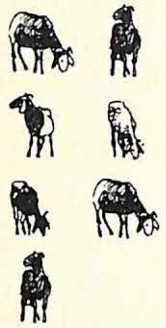
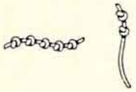
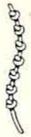
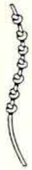
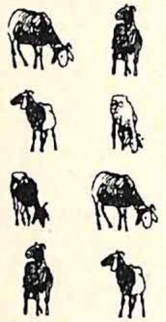
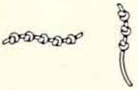
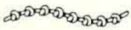
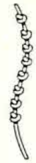
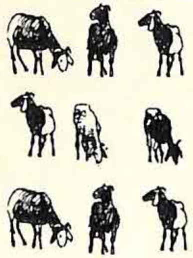
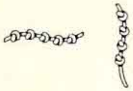
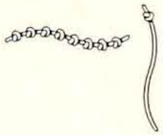
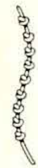
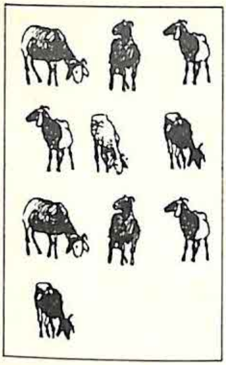
They go on.

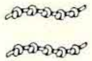
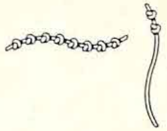
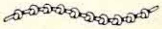
	Tani says	Jacob says	Ali says
	 one cord, two knots	 seven knots	 seven knots
	 one cord, three knots	 one cord	 eight knots

Now Jacob has a full cord. He does not need a new number name. He says *one cord*. Ali, however, has to use yet another new number name. He says *eight*.

More goats are recorded.

	Tani says	Jacob says	Ali says
	 one cord, four knots	 one cord, one knot	 nine knots



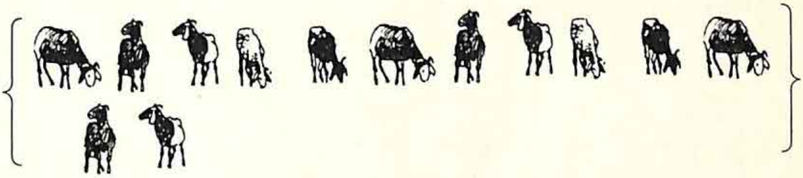
Tani says	Jacob says	Ali says
 two cords	 one cord, two knots	 one cord

Now Ali has a full cord. Like Tani and Jacob he needs no more new number names. We see that, at the same time, Tani has now filled another cord.

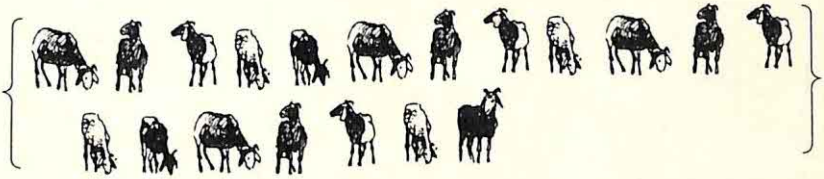
Exercises

1. Show how Tani, Jacob, and Ali each record the following. Also, in each case, write the number names which they say.

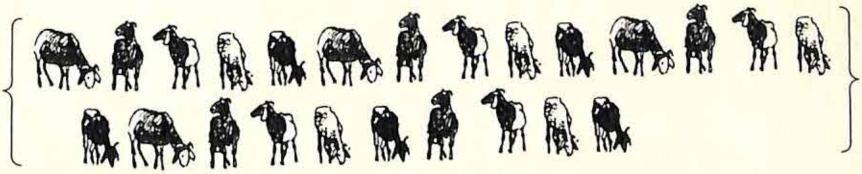
(a)



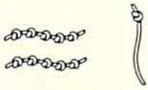
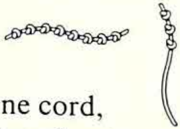
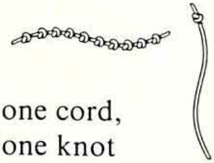
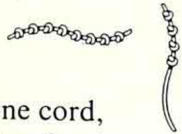
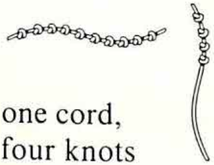
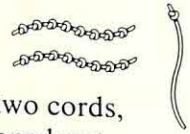
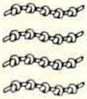
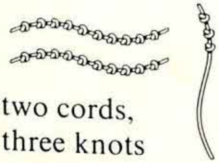
(b)



(c)

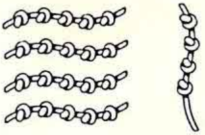


2. In (a) below we see how Tani, Jacob, and Ali each record the same number using knots and cords. We also see the number name each says. Copy the table and fill in the spaces.

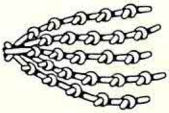
	Tani says	Jacob says	Ali says
(a)	 two cords, one knot	 one cord, three knots	 one cord, one knot
(b)		 one cord, four knots	
(c)			 one cord, four knots
(d)		 two cords, one knot	
(e)	 four cords		
(f)			 two cords, three knots

SPOKEN NUMBER NAMES

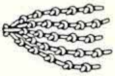
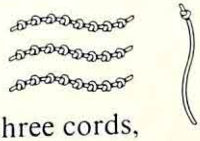
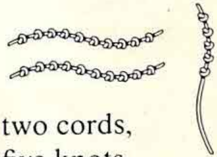
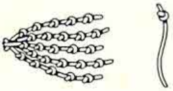
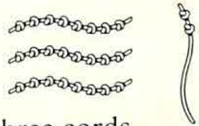
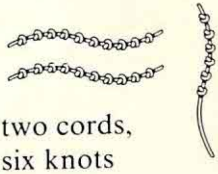
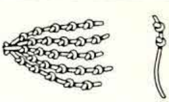
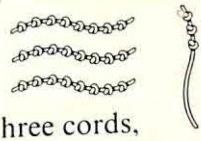
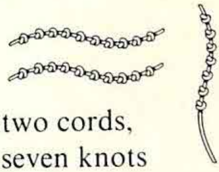
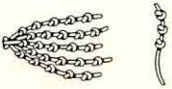
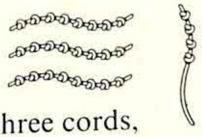
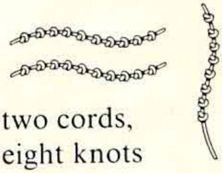
In exercise 1(c) we see that Tani records the goats by:



and says *four cords, four knots*. For the next goat he ties another knot and now has another full cord. This, of course, gives him a bundle.

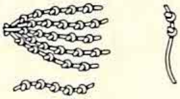
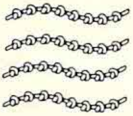
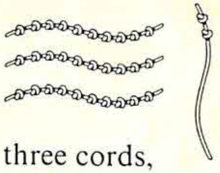
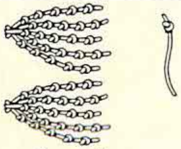
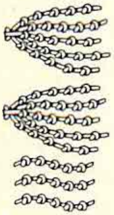
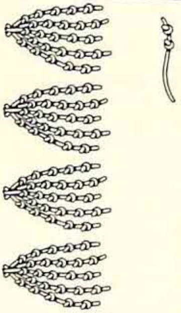


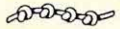
He now says that he has one bundle. As more goats are recorded the cords for the three men look like this.

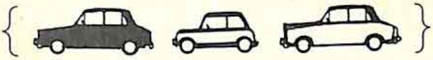
Tani says	Jacob says	Ali says
 one bundle	 three cords, one knot	 two cords, five knots
 one bundle, one knot	 three cords, two knots	 two cords, six knots
 one bundle, two knots	 three cords, three knots	 two cords, seven knots
 one bundle, three knots	 three cords, four knots	 two cords, eight knots

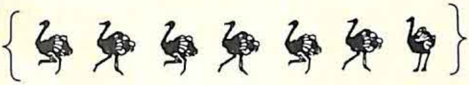
Exercises

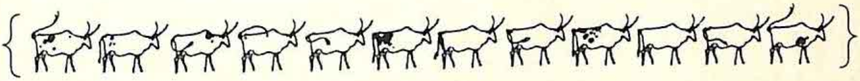
3. In (a) below we see how Tani, Jacob, and Ali each record the same number using knots and cords. We also see the number name each says. Copy the table and fill in the spaces.

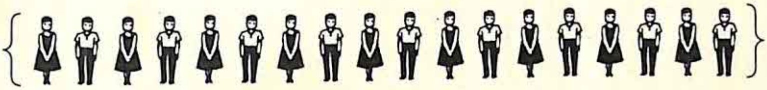
	Tani says	Jacob says	Ali says
(a)	 one bundle, one cord, two knots	 four cords	 three cords, two knots
(b)	 two bundles, one knot		
(c)	 two bundles, three cords		
(d)	 four bundles, two knots		

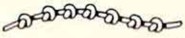
4. Ahmed's full cord is like this: 
Write, in words, how he would say the number of:

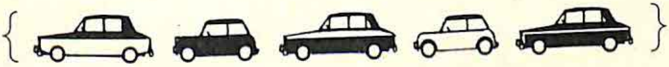
(a) 

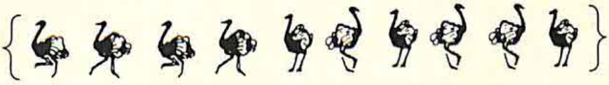
(b) 

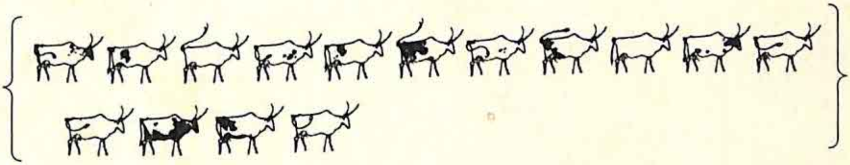
(c) 

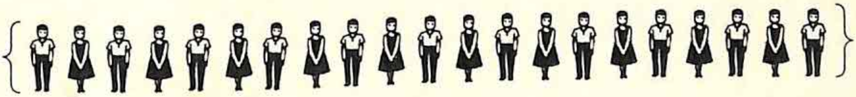
(d) 

5. John's full cord is like this: 
Write, in words, how he would say the number of:

(a) 

(b) 

(c) 

(d) 

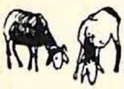
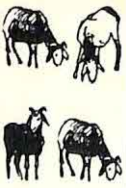
Important ideas in this chapter

1. Spoken number names for knots.
2. Spoken number names for cords and knots.
3. Spoken number names for bundles, cords, and knots.

4. NUMERALS

A. Numerals for cords and knots

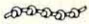
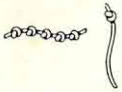
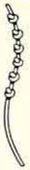
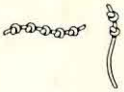
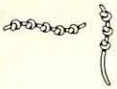
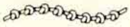
In this chapter we again move forward many years. Once more we have three men recording the numbers of their goats. Fortunately they too are called Tani, Jacob, and Ali. They still use knots and say the number names as before. They have also begun to keep a record by making marks on stone and in clay. These marks we call numerals. They are not like the numerals we now use. However, for simplicity, we shall use present-day numerals to show what Tani, Jacob, and Ali write. The pictures below show the numerals for the first four numbers.

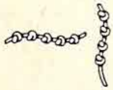
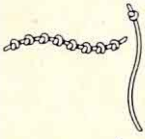
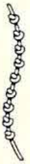
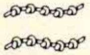
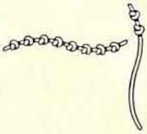
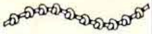
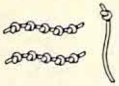
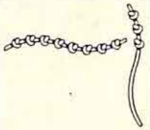
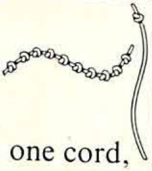
	Tani		Jacob		Ali	
		Knots		Knots		Knots
	 one knot	1	 one knot	1	 one knot	1
	 two knots	2	 two knots	2	 two knots	2
	 three knots	3	 three knots	3	 three knots	3
	 four knots	4	 four knots	4	 four knots	4

When we come to the next number Tani has one cord. To show this he uses '1' again. Of course, he must not put it under 'knots'. He must have another column—the 'cords' column. He shows the number like this.

Cords	Knots
1	

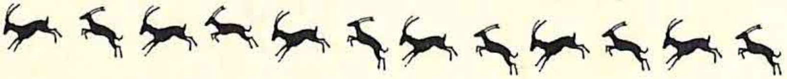
The following pictures show how Tani, Jacob, and Ali represent the next few numbers.

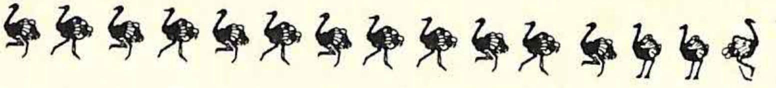
Tani			Jacob			Ali		
	Cords	Knots		Cords	Knots		Cords	Knots
 one cord	1		 five knots		5	 five knots		5
 one cord, one knot	1	1	 six knots		6	 six knots		6
 one cord, two knots	1	2	 seven knots		7	 seven knots		7
 one cord, three knots	1	3	 one cord	1		 eight knots		8

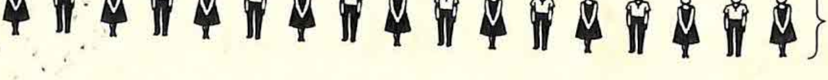
Tani			Jacob			Ali		
	Cords	Knots		Cords	Knots		Cords	Knots
 one cord, four knots	1	4	 one cord, one knot	1	1	 nine knots		9
 two cords	2		 one cord, two knots	1	2	 one cord	1	
 two cords, one knot	2	1	 one cord, three knots	1	3	 one cord, one knot	1	1

Exercises

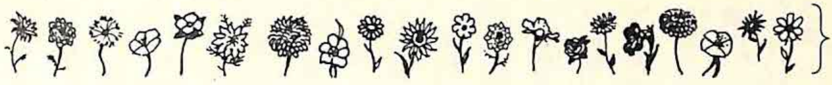
1. Show how Tani, Jacob, and Ali use numerals and columns to represent the numbers of:

(a) {  }

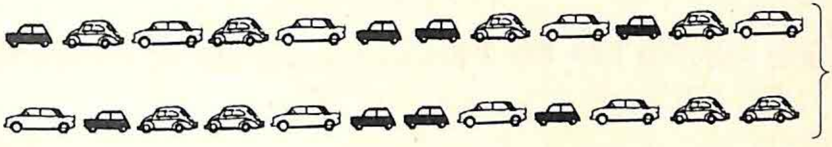
(b) {  }

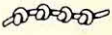
(c) {  }

(d)



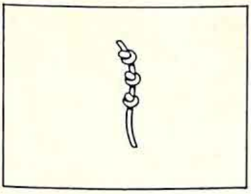
(e)



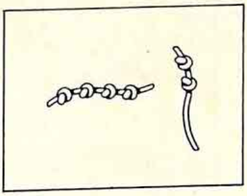
2. Here is a full cord for Ahmed. 

(a) Show how he uses numerals and columns to represent:

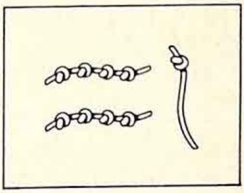
(i)



(ii)



(iii)



(b) Draw Ahmed's cords and knots for the following numbers which he has written:

(i)

Cords	Knots
2	3

(ii)

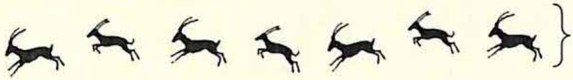
Cords	Knots
3	1

(iii)

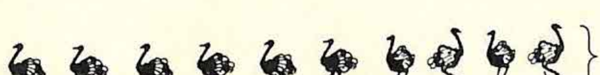
Cords	Knots
2	

(c) Show how he uses numerals and columns to represent the number of:

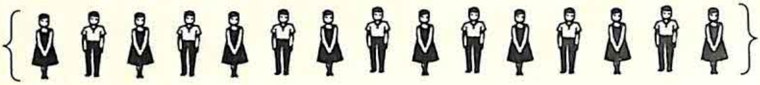
(i)

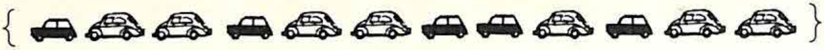


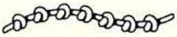
(ii)



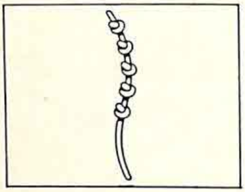
S.C.E.R.T., West Bengal
Library
Calcutta
25/3, B. C. Road

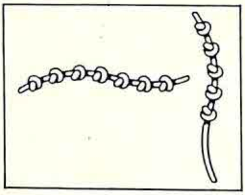
(iii) 

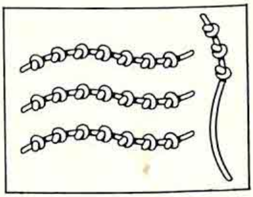
(iv) 

3. Here is a full cord for John. 

(a) Show how he uses numerals and columns to represent:

(i) 

(ii) 

(iii) 

(b) Draw John's cords and knots for the following numbers which he has written:

(i)

Cords	Knots
1	2

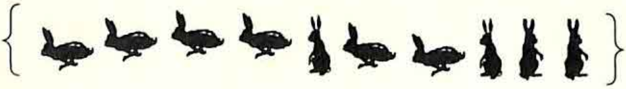
(ii)

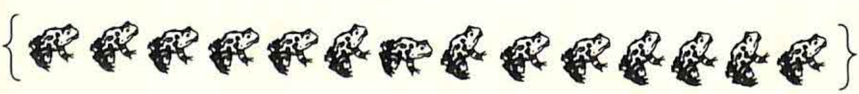
Cords	Knots
3	

(iii)

Cords	Knots
4	6

(c) Show how he uses numerals and columns to represent the number of:

(i) 

(ii) 

(iii)

(iv)

In our work so far we have been writing numbers using the headings 'cords' and 'knots'. This takes time. Usually it is not necessary to have headings. If Tani writes down

3 2

(and says 'three two'), we know that he has three cords and two knots.

Tani has to be careful, however, when he wants to show that he has just three cords. With headings he writes

Cords	Knots
3	

Without headings he must do more than write

3

We would think that this represents three knots rather than three cords. He must show that there are no knots. Using columns, he can do this by putting 0 under 'knots'.

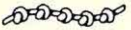
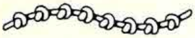
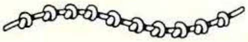
Cords	Knots
3	0

Without columns he writes the number as

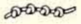
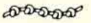
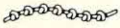
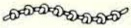
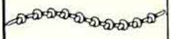
3 0

Exercises

4. In the first row of the table below, Tani records a number as 24. Jacob and Ali record the same number as 16 and 14 respectively. Copy the table and fill in the spaces in the other rows.

	Tani	Jacob	Ali
Full cord			
(a)	24	16	14
(b)	30		
(c)		20	
(d)			20
(e)		15	
(f)	14		

5. In the first row of the table below, Ahmed records a number as 21. The other men record the same number as shown. Copy the table and fill in the spaces in the other rows.

	Ahmed	Tani	John	Jacob	Ali
Full cord					
(a)	21	14	12	11	9
(b)		30			
(c)					10
(d)			16		
(e)		13			

B. Numerals for bundles, cords, and knots

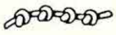
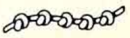
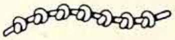
So far each of the men has recorded only cords and knots. We see, however, that they can easily record larger numbers using only the same numerals. For example, Tani records two bundles, three cords, one knot as

231

He is still making use of columns but does not need headings.

Exercises

6. Copy and complete the following table:

	Ahmed	Tani	John
Full cord			
(a)	121		
(b)			45
(c)		31	
(d)			111
(e)	312		

7. Draw one of Tani's full cords. Write the set of numerals which he uses. Do the same for Jacob, Ali, Ahmed, and John. What do you notice?

C. Bases

From our work so far we see that each man has a different number of knots on a full cord. We also see that for each of them the number of cords in a bundle is the same as the number of knots on a full cord.

Nowadays we say that, for example, Tani is using base five. At the same time:

Ahmed is using base four.
John is using base seven.
Jacob is using base eight.
Ali is using base ten.

Because we have ten fingers it is natural for us to use base ten as Ali does. In the past some people have used the fingers of one hand only and have worked in base five. Others have used their toes as well as their fingers and have worked in base twenty. Throughout the world today, however, ten is the base which is most often used.

Exercises

8. Give the set of numerals which are used for:
   (a) base ten,
   (b) base three,
   (c) base six,
   (d) base nine.

9. Are the numerals which we normally use sufficient for work in base twelve? If not, what must we do?

10. What is the smallest possible base? Give the set of numerals which it requires.

Ali, like us, works in base ten. The headings of his columns are 'bundles', 'cords', and 'knots'. We use columns in the same way, but our headings are 'hundreds', 'tens', and 'units'. We show these as

H	T	U

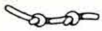
Using these headings we can represent numbers only up to 999. For the numbers which follow we need another column. The number after 999 is a thousand. The new column is called the 'thousands' column. We show a *thousand* by writing 1000. With more columns we can represent every number using only the numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

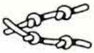
Exercise

11. Each number is in base ten. What is the value of the red numeral in:
   (a) 235?
   (b) 576?
   (c) 129?
   (d) 444?
   (e) 1769?
   (f) 3415?
   (g) 2678?
   (h) 3094?

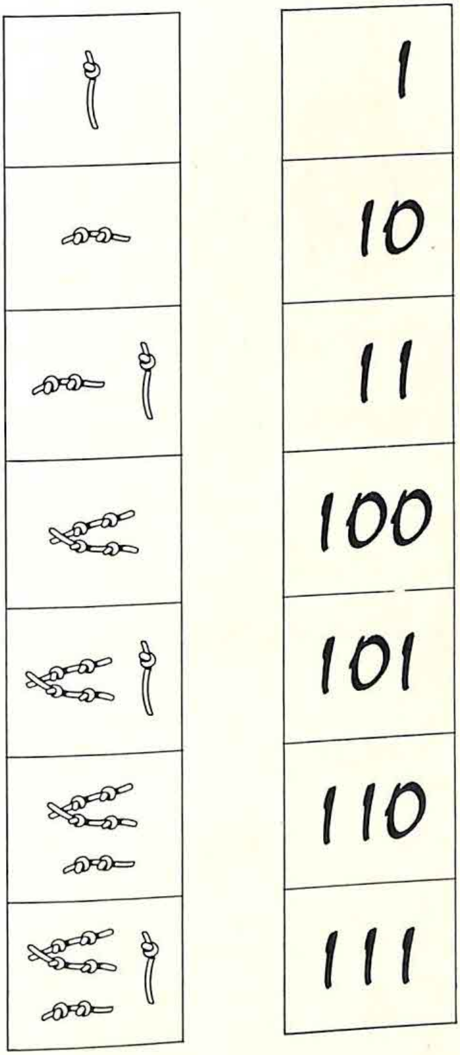
In exercise 11(d) we see that the numeral '4' appears in three different columns or places. The value of each 4 depends on its place. The use of place-value in this way enables us to write numbers using only a small set of numerals.

D. Base two

While we usually work in base ten there is another base which is becoming increasingly important. This is base two. In this base a full cord looks like this: 

and a bundle like this: 

The pictures below show how numerals are used for the first few numbers in base two.



From these pictures we see that in this base, 0 and 1 are the only numerals we need to use.

Here we show the first few numbers written both in base ten and in base two:

Base ten
1
2
3
4
5
6
7

Base two
1
10
11
100
101
110
111

In base ten the next number is 8. To show this in base two we need another column. We write:

1000

In base ten the next numbers are 9 and 10. In base two they are written as 1001 and 1010.

Exercise

12. The following numbers are written in base ten. Write them in base two.

(a) 11
(b) 12
(c) 13
(d) 14

We have seen that the column headings for base ten are 'units', 'tens', 'hundreds', 'thousands', etc. For base two the headings are 'units', 'twos', 'fours', 'eights', 'sixteens', etc. Here are some numbers written in base ten and in base two with these column headings.

Base ten			
Thousands	Hundreds	Tens	Units
			1
			2
			3
			4
			7
			8
		1	5
		1	6
		4	3
		5	0
		6	3

	Base two				
Thirty-twos	Sixteens	Eights	Fours	Twos	Units
					1
				1	0
				1	1
			1	0	0
			1	1	1
		1	0	0	0
		1	1	1	1
	1	0	0	0	0
1	0	1	0	1	1
1	1	0	0	1	0
1	1	1	1	1	1

Exercises

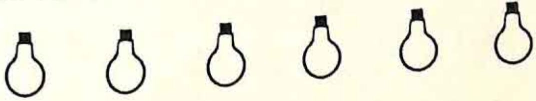
13. The following numbers are written in base two. Write them in base ten.

(a) 101
(b) 110
(c) 1010
(d) 1001
(e) 1110
(f) 11011
(g) 11001
(h) 100001
(i) 100101
(j) 110100
(k) 111111
(l) 1000000

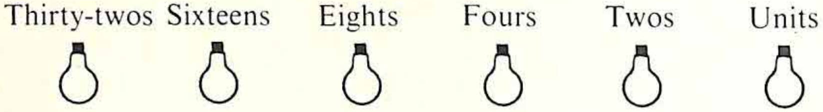
14. The following numbers are written in base ten. Write them in base two.

(a) 5
(b) 6
(c) 9
(d) 10
(e) 20
(f) 30
(g) 49
(h) 70
(i) 100

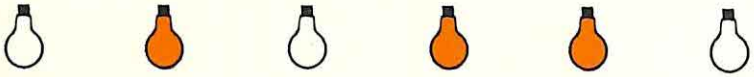
Base two is important because, in this base, any number can be written using only the symbols 0 and 1. We can go further. Instead of the written symbols 0 and 1, we can use other means of showing a number. For example, we can use a row of electric light bulbs, like this:



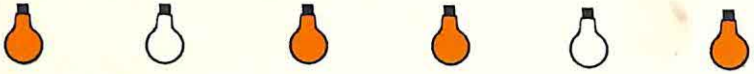
Each bulb can, of course, be switched 'on' or switched 'off'. We can say that when it is 'on' it represents 1 and when it is 'off' it represents 0. If now we put the bulbs under the column headings for base two we have:



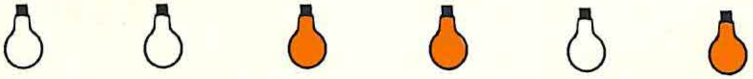
We can now show a number such as 10110 by switching on the lights in the 'twos', 'fours', and 'sixteens' columns:



Another number, for example 101101, is shown as:



To use these bulbs to show numbers written in other bases we first change the numbers to base two and then proceed as before. For example, 13 (base ten) is 1101 (base two), so we show it with the bulbs as:



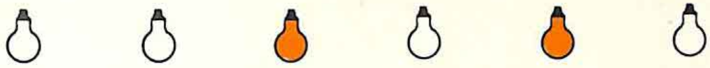
Here are some more examples.

Base ten	Base two					
	Thirty-twos	Sixteens	Eights	Fours	Twos	Units
1	○	○	○	○	○	●
2	○	○	○	○	●	○
3	○	○	○	○	●	●
4	○	○	○	●	○	○
7	○	○	○	●	●	●
8	○	○	●	○	○	○
15	○	○	●	●	●	●
16	○	●	○	○	○	○
43	●	○	●	○	●	●

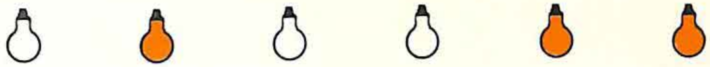
Exercises

15. Write, in base two and then in base ten, the numbers represented by:

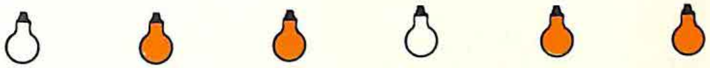
(a)



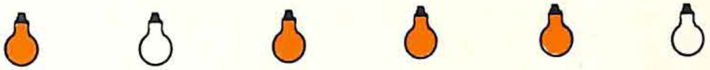
(b)



(c)



(d)



16. Show, by a drawing, how the following numbers (base two) are represented by a set of light bulbs.

(a) 101	(c) 1110	(e) 101111
(b) 1001	(d) 10101	(f) 111100

17. Show how the following numbers (base ten) are represented by a set of light bulbs.

(a) 6	(c) 17	(e) 41
(b) 11	(d) 29	(f) 57

In the modern computer the representation of 1 and 0 depends upon the presence or absence of an electrical current or an electrical charge. Details cannot be given in this book but many texts are now available.

Important ideas in this chapter

1. The written symbols (numerals) for knots.
2. Using the numerals for cords and knots.
3. Using the numerals for bundles, cords, and knots.
4. The use of 0.
5. The idea of a base.
6. Base ten: hundreds, tens, and units.
7. Place-value.
8. Base two.

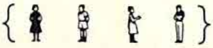
5. TOWARDS A NUMBER LINE

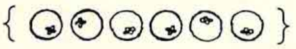
A. Numbers in order

Let us pause for a moment and look back at the ideas we have discussed so far. We first looked at sets and saw how one-to-one correspondence led us to the idea of a number. We then saw how man first recorded numbers by using practical aids. Next we looked at the number names and numerals. Finally we saw how by using the idea of place-value man was able to record all numbers using only the numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

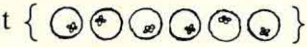
We notice that as we have progressed the numbers have naturally fallen into order. We found ourselves saying 'one, two, three, four, etc.' We now look at this order rather more carefully.

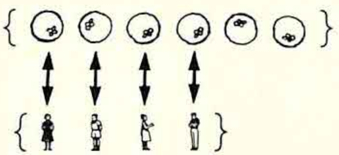
We remember that the number of

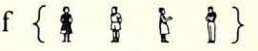
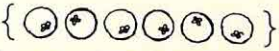
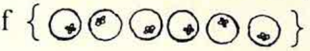
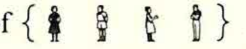
 is 4,

 is 6,

is 1, etc.

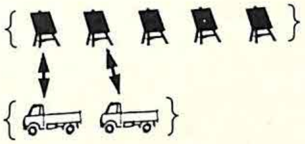
When we try to put  and into one-to-one correspondence we get:

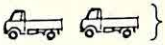


We see that all the members of  are matched but that there are some unmatched members in . For this reason we say that the number of  is greater than the number of .

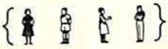
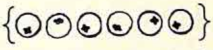
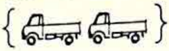
This relationship is true, of course, whatever members are in the sets. For example, the number of {△ ▽ ▷ △ ▽ ◁} is greater than the number of {□ □ ▫ □}; the number of { } is greater than the number of { }.

In the same way the matching



shows us that the number of { } is greater than the number of {}.

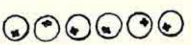
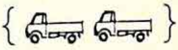
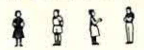
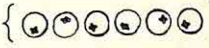
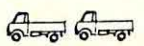
We can also show these relationships as:

the number of {} is less than the number of {};
the number of {} is less than the number of { }.

To write these statements we usually use the symbols:

> for 'is greater than',
< for 'is less than'.

Using these symbols the statements are written as:

the number of {} > the number of { }
the number of { } > the number of {}
the number of {} < the number of {}
the number of {} < the number of { }

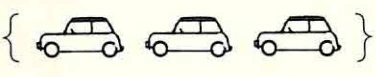
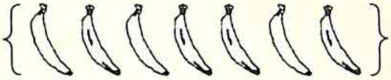
Using numerals we write:

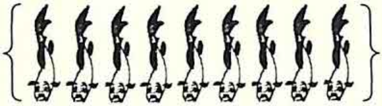
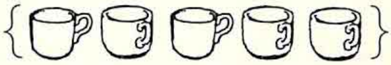
$6 > 4$ or $4 < 6$

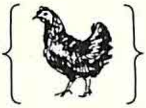
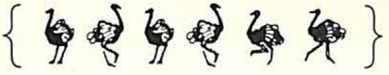
$5 > 2$ or $2 < 5$

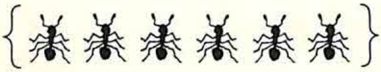
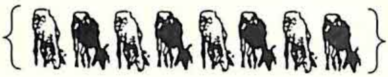
Exercises

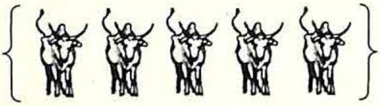
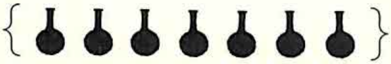
1. Show, in both ways, the relationship between the numbers of:

(a)  and 

(b)  and 

(c)  and 

(d)  and 

(e)  and 

2. Use the symbols $<$ and $>$ to show the relationship between:

(a) 7, 2
(b) 3, 1
(c) 4, 0
(d) 5, 8
(e) 4, 3
(f) 3, 5
(g) 0, 7
(h) 2, 5
(i) 99, 100

3. Because $1 < 3$ and $3 < 8$ we can write $1 < 3 < 8$. Show, in this way, the relationship between:

(a) 2, 4, 7
(b) 1, 5, 6
(c) 7, 3, 10
(d) 5, 9, 6
(e) 9, 11, 10
(f) 6, 5, 4

4. Using the symbol $<$, write in order:

(a) 4, 1, 3, 2
(b) 10, 7, 9, 8
(c) 6, 10, 9, 7, 8
(d) 6, 8, 5, 9, 7
(e) 2, 4, 5, 3, 1
(f) 2, 4, 3, 1, 6, 5, 9, 7, 8, 10

These exercises show us why we put numbers in the order 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, etc. In everyday life we make use of this order when we count. For this reason we often call {1, 2, 3, 4, 5, . . .}, the set of counting numbers.

We also use our numbers in this order when, for example, we 'number' the pages of a book. In this book the pages of the first chapter are numbered 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. The number of the last page of the chapter is 10. Here the numeral 10 on the page tells us:

(a) that there are ten pages in the first chapter,

(b) that the previous page number is 9 and the next is 11.

The '10' now serves two purposes. It tells us the number of the set of pages in the chapter. It also tells us the position of the page; it is the 'tenth' page.

Here are some numbers being used in the first way:

(i) There are twenty-six children in the class.

(ii) There are fourteen crates on the lorry.

(iii) Five people got on the bus at the last stop.

Here are some examples where order is shown:

(iv) Hassan was fifth in the examination.

(v) You are now reading chapter five.

(vi) Mary was born in 1947.

In (i), (ii), and (iii) we say that we are using the number in the cardinal way. In (iv), (v), and (vi) we say that we are using the number in the ordinal way.

Exercises

5. Look at the numbers in the following paragraph. List those which are used in (a) the cardinal way, (b) the ordinal way.

   'Volcanic islands are found all over the world. Perhaps the most spectacular was Krakatoa, in the Sunda Strait between Java and Sumatra. On 27th August, 1883, Krakatoa exploded. After a series of eruptions going on for 2 days, there was almost nothing left of the island which had been 1400 feet above sea level. A 100-foot tidal wave raced along the strait killing tens of thousands of people. The sound of the explosion was heard 3000 miles away.'

   New Knowledge, vol. 4

6. Write a sentence to show the use of each of the following numbers in (a) the cardinal way, (b) the ordinal way.

   365; 4; 7; 12; 1,000,000.

We finish this section by looking again at the empty set. Because there are fewer members in { } than in { }, { }, and { } we say that

$$0 < 1 < 2 < 3.$$

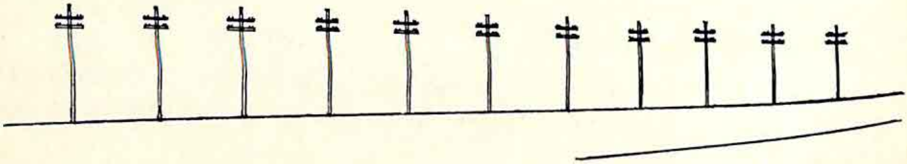
In fact, 0 is less than any of the other numbers. So when we include 0 and write the numbers in order, we have:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, etc.

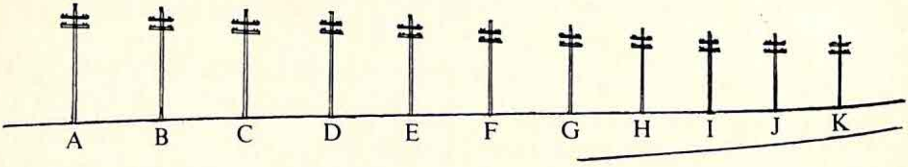
This set of numbers—which goes on and on—we call the natural numbers.

B. Naming points on a line

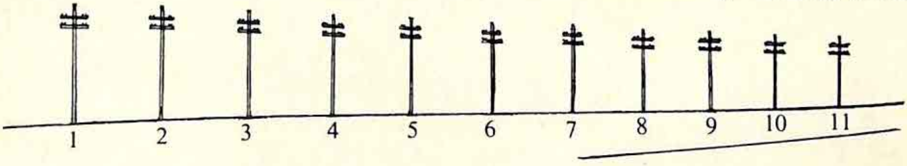
Here is a picture of some telegraph poles on a straight road.



From time to time repairs have to be done and it is necessary to be able to tell the workmen which poles need attention. This can be done by using the letters *A*, *B*, *C*, *D*, etc.

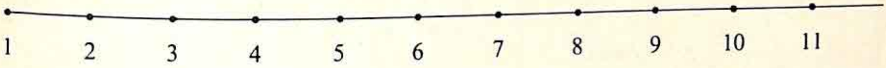


Of course we shall soon use up all the letters. It is simpler to use numbers as there will always be enough of them. We number the poles like this:



Each pole now has a number and because the numbers are in order we can quickly find a particular pole.

Instead of drawing a picture we can simply show the road as a line and the poles by points on the line, like this:



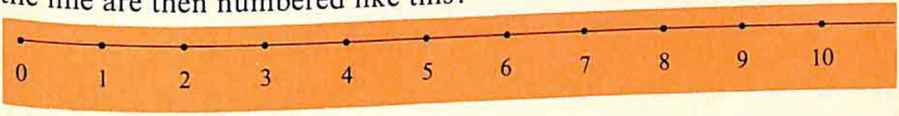
Exercise

7 Draw a line numbered from 1 to 20 and show the poles with numbers:

(a) 5, 9, 15
(b) between 7 and 12
(c) less than 14
(d) greater than 9
(e) from 6 to 16 inclusive
(f) greater than 4 but less than 13

In using a number line in the way described above we have been thinking of the numbers in the ordinal sense. We have been interested only in the positions of the poles. We could, however, use the same line to give the distance of each pole from the first pole.

In order that the number of a pole shall tell us its distance from the first pole, we must use the number '0' for the first pole. The points on the line are then numbered like this:



Using this method we see that, for example, the distance of the pole numbered '7' from the first pole is 7 spaces. In the same way the distance of the pole numbered '0' is 9 spaces.

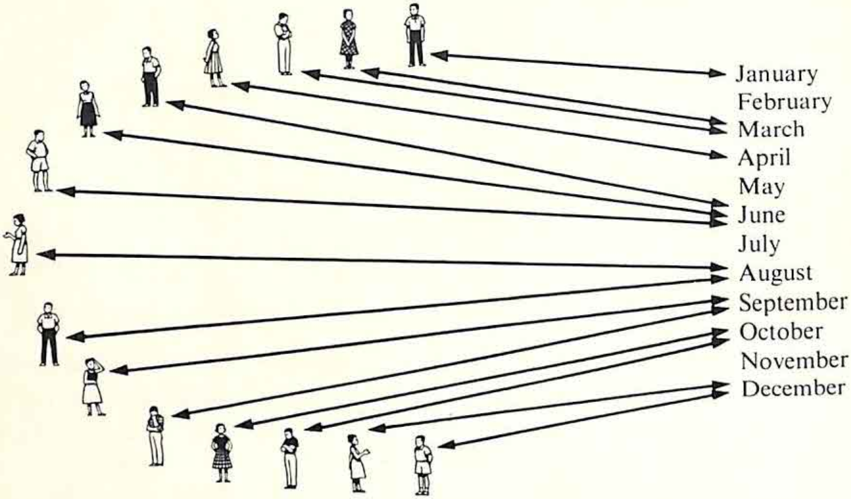
So far we have a number line to show the position of telegraph poles along a road. This is but one of its many uses. As we progress we shall find that there are more and more applications of the idea.

Important ideas in this chapter

1. The idea of 'greater than' and 'less than'.
2. The idea of numbers being used in (a) a cardinal way, (b) an ordinal way.
3. The use of the phrase 'natural numbers' to describe $\{0, 1, 2, 3, 4, \ldots\}$.
4. The idea of a number line.

6. USING A NUMBER LINE

In chapter 1 (exercise 2, page 4) we showed, with a diagram, a set of people and the months in which they were born. The diagram is shown here again.



We can show this information in another way like this:

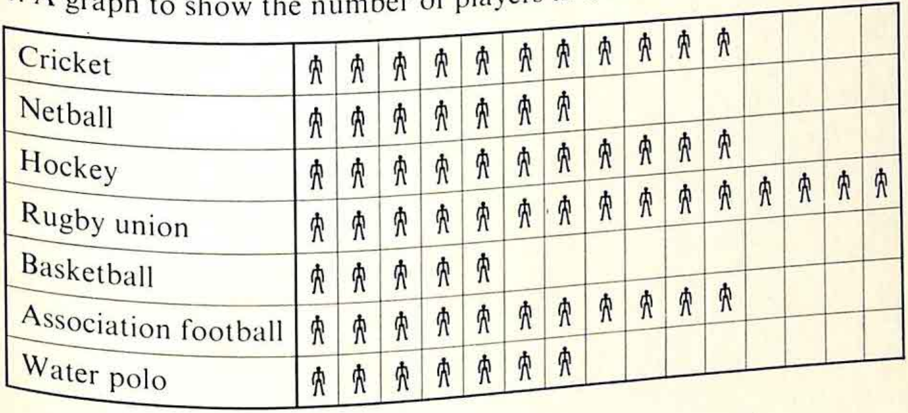
January	🧍
February	
March	🧍 🧍
April	🧍
May	
June	🧍 🧍 🧍
July	
August	🧍 🧍
September	🧍 🧍
October	🧍 🧍
November	
December	🧍 🧍

From the diagram we can easily go on to list the number of people born in each month, like this:

Month	Number of people
January	1
February	0
March	2
April	1
May	0
June	3
July	0
August	2
September	2
October	2
November	0
December	2

Exercises

1. A graph to show the number of players in a team.



Write down all the information you can get from this graph.

2. A graph to show the traffic along a road in a period of ten minutes.

Cars	🚗	🚗	🚗	🚗	🚗	🚗		
Buses	🚌	🚌						
Bicycles	🚲	🚲	🚲	🚲	🚲	🚲	🚲	🚲
Lorries	🚚	🚚	🚚					
Vans	🚐	🚐						
Motor-bikes	🏍	🏍	🏍	🏍				

Write down all the information you can get from this graph.

3. This table shows how 35 people get to their office. Show this information by a graph.

Method of transport	Number of travellers
Bicycle	12
Bus	7
Car	2
Motor-bike	4
On foot	10

4. This graph shows a firm's morning mail. Each picture represents 10 items. How many (a) letters, (b) postcards, (c) circulars, (d) parcels, did they receive?

Letters	✉	✉	✉	✉	✉	✉	✉
Postcards	▭	▭	▭	▭	▭		
Circulars	📰	📰					
Parcels	📦	📦	📦				

5. Here is the number of items in a firm's morning mail.

Letters	80
Postcards	50
Circulars	20
Parcels	40

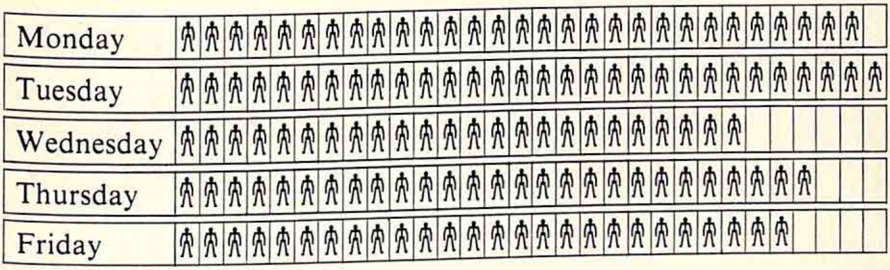
Show this information by a graph (using one picture for 10 items).

6. On another morning the mail is:
letters 65; postcards 45; circulars 17; parcels 23.
Show this information as well as you can by a graph (using one picture for 10 items).

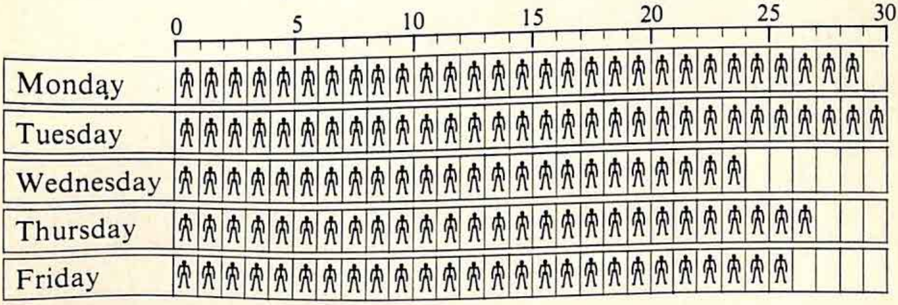
7. Count the number of vehicles passing along a busy road in ten minutes. Show, by a graph, the information you obtain.

8. Find the size of the sandals (or shoes) of several of your friends. Show the information by a graph.

This graph shows the attendance in a class of 30 for each day of a week.

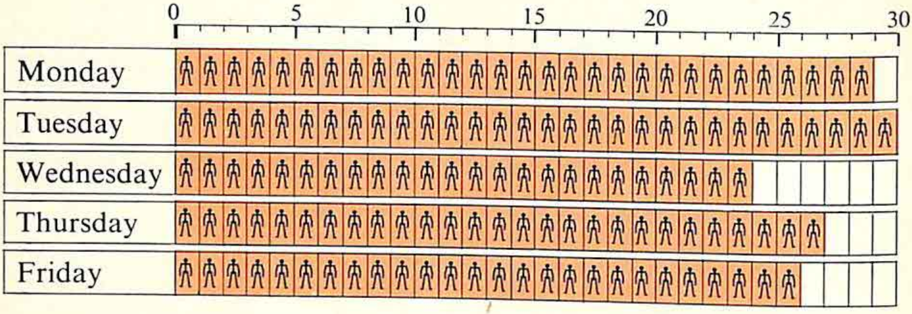


This is not a very easy graph to read. The counting takes too much time. To make it easier we put a number line along the top, like this.

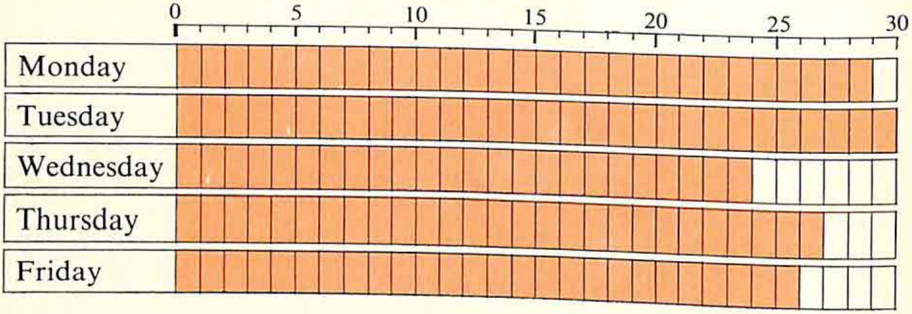


With this guide it is easy for us to read the graph quickly.

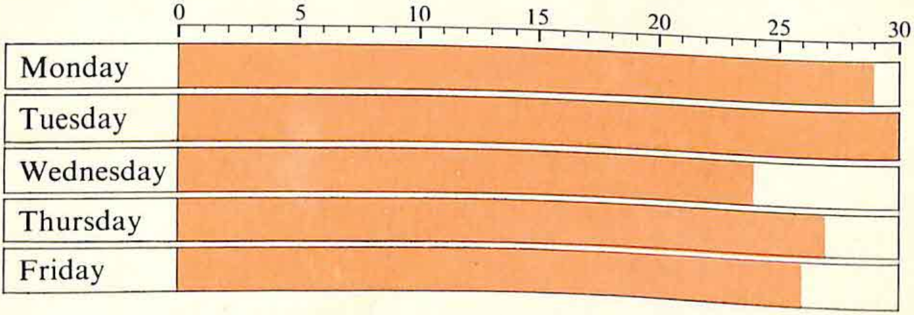
The graph is also made easier to read if we use colour like this.



We now see that we can show the information just as well by using the coloured blocks only. The pictures are no longer necessary. The graph now looks like this.



There is, of course, no need to put in all the small lines. We can simply colour one large block for each day of the week, like this.



We call this kind of graph a block graph. The picture graphs we used earlier we call isotypes.

Exercises

9. Draw a block graph to show the information given in:
    (a) Exercise 1 (page 49).
    (b) Exercise 2 (page 50).
    (c) Exercise 3 (page 50).
    (d) Exercise 4 (page 50).
    (e) Exercise 5 (page 51).
    (f) Exercise 6 (page 51).
10. List the month of birth of several of your friends. Show the information by a block graph.
11. List the age of several of your friends. Show the information by a block graph.
12. Look at these two paragraphs.

> 'Mathematics itself is changing both to meet the needs of today and to make possible the developments of tomorrow. New ideas have emerged which help to unify aspects of the subject which formerly seemed unrelated to each other. At the frontiers of knowledge, modern mathematics has made the space programme possible. It may be that some of these ideas will prove to be of value in the work of the primary school.
>
> For many years the teaching of traditional arithmetic was kept in its place, one period a day, very frequently time-tabled in the first half of the morning. It may be that the time is now ripe for the emancipation of arithmetic, for it to take its rightful place in the wider field of mathematics.'
>
> *Nuffield Guide, 'I do and I understand'.*

Count the number of words with one letter; two letters; three letters; four letters; etc. Show the results by a block graph. Write down all the information you can get from this graph.

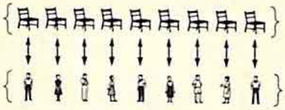
Important ideas in this chapter

1. Using pictures for showing information.
2. The use of isotypes.
3. The use of block graphs.

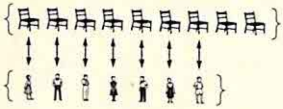
ANSWERS

1. ONE-TO-ONE

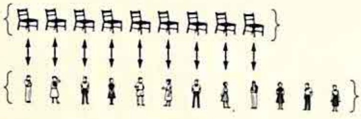
4. (a)



(b)



(c)



5. (b), (f), (g).

6. (a) *E*, *K*.

   (b) *F*, *G*.

   (c) Set *B*: *I*.
   Set *D*: *H*, *J*.

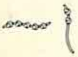
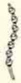
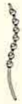
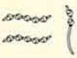
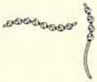
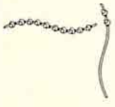
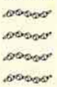
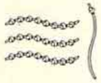
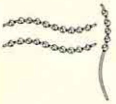
2. RECORDING BEFORE COUNTING

1. There is a one-to-one correspondence between the set of stones and the set of goats.

2. There is a one-to-one correspondence between the set of cuts and the set of goats.

3. The sticks can be carried from place to place more easily than the stones.

5.

	Tani	Jacob	Ali
(a)			
(b)			
(c)			

6.

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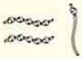
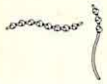
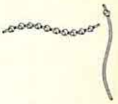
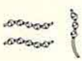
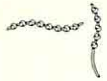
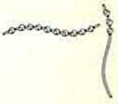
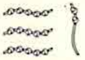
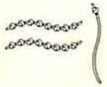
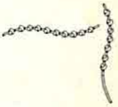
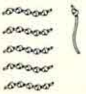
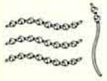
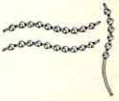
(a)			
(b)			
(c)			
(d)			

7.

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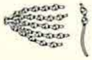
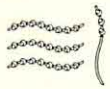
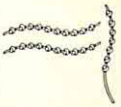
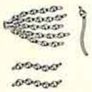
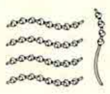
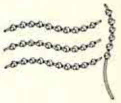
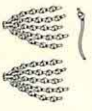
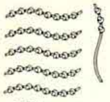
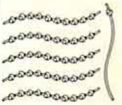
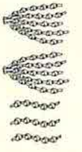
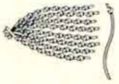
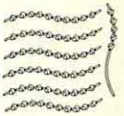
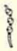
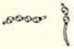
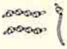
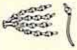
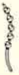
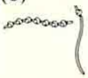
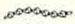
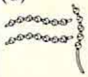
(a)			
(b)			
(c)			
(d)			

ANSWERS

8.

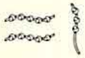
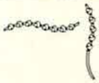
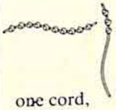
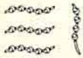
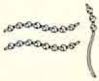
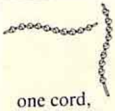
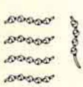
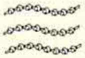
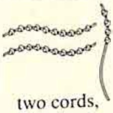
(a)	(b)	(c)	(d)
			

9.

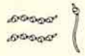
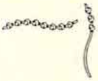
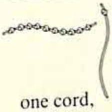
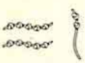
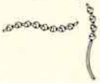
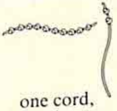
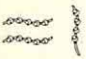
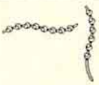
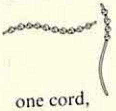
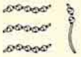
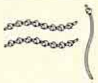
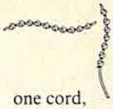
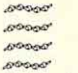
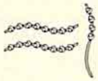
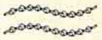
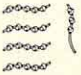
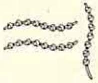
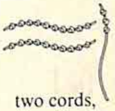
(a)	(b)	(c)	(d)
			

3. SPOKEN NUMBER NAMES

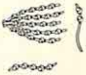
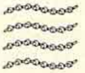
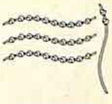
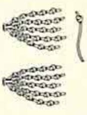
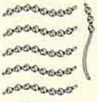
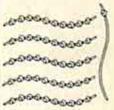
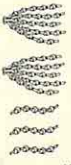
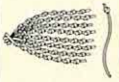
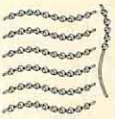
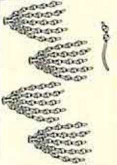
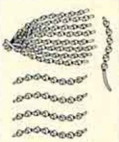
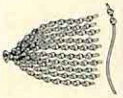
1.

	Tani says	Jacob says	Ali says
(a)	 two cords, three knots	 one cord, five knots	 one cord, three knots
(b)	 three cords, four knots	 two cords, three knots	 one cord, nine knots
(c)	 four cords, four knots	 three cords	 two cords, four knots

2.

	Tani says	Jacob says	Ali says
(a)	 two cords, one knot	 one cord, three knots	 one cord, one knot
(b)	 two cords, two knots	 one cord, four knots	 one cord, two knots
(c)	 two cords, four knots	 one cord, six knots	 one cord, four knots
(d)	 three cords, two knots	 two cords, one knot	 one cord, seven knots
(e)	 four cords	 two cords, four knots	 two cords
(f)	 four cords, three knots	 two cords, seven knots	 two cords, three knots

3.

	Tani says	Jacob says	Ali says
(a)	 one bundle, one cord, two knots	 four cords	 three cords, two knots
(b)	 two bundles, one knot	 six cords, three knots	 five cords, one knot
(c)	 two bundles, three cords	 one bundle, one knot	 six cords, five knots
(d)	 four bundles, two knots	 one bundle, four cords, six knots	 one bundle, two knots

4. (a) Three knots.
   (b) One cord, three knots.
   (c) Three cords.
   (d) One bundle, two knots.

5. (a) Five knots.
   (b) One cord, three knots.
   (c) Two cords, one knot.
   (d) Three cords.

4. NUMERALS

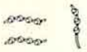
1.

	Tani		Jacob		Ali	
	Cords	Knots	Cords	Knots	Cords	Knots
(a)	2	2	1	4	1	2
(b)	3	0	1	7	1	5
(c)	3	2	2	1	1	7
(d)	4	0	2	4	2	0
(e)	4	4	3	0	2	4

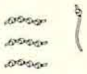
2. (a)

	Cords	Knots
(i)		3
(ii)	1	2
(iii)	2	1

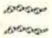
(b) (i)



(ii)



(iii)



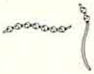
(c)

	Cords	Knots
(i)	1	3
(ii)	2	2
(iii)	3	3
(iv)	3	0

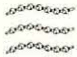
3. (a)

	Cords	Knots
(i)		5
(ii)	1	5
(iii)	3	3

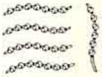
(b) (i)



(ii)



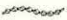
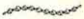
(iii)



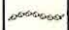
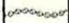
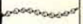
(c)

	Cords	Knots
(i)	1	3
(ii)	2	0
(iii)	2	3
(iv)	1	6

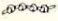
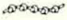
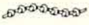
4.

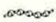
	Tani	Jacob	Ali
Full cord			
(a)	24	16	14
(b)	30	17	15
(c)	31	20	16
(d)	40	24	20
(e)	23	15	13
(f)	14	11	9

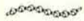
5.

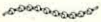
	Ahmed	Tani	John	Jacob	Ali
Full cord					
(a)	21	14	12	11	9
(b)	33	30	21	17	15
(c)	22	20	13	12	10
(d)	31	23	16	15	13
(e)	20	13	11	10	8

6.

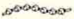
	Ahmed	Tani	John
Full cord			
(a)	121	100	34
(b)	201	113	45
(c)	100	31	22
(d)	321	212	111
(e)	312	204	105

7. Tani  {0, 1, 2, 3, 4}

Jacob  {0, 1, 2, 3, 4, 5, 6, 7}

Ali  {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

Ahmed {0, 1, 2, 3}

John  {0, 1, 2, 3, 4, 5, 6}

The number of each man's set of numerals is the same as the number of knots on his full cord.

8. (a) {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}
   (b) {0, 1, 2}
   (c) {0, 1, 2, 3, 4, 5}
   (d) {0, 1, 2, 3, 4, 5, 6, 7, 8}

9. No. We have to invent two more numerals, one for 'ten' and one for 'eleven

10. Base two. {0, 1}

11. (a) Three tens
    (b) Six units
    (c) One hundred
    (d) Four tens
    (e) Seven hundreds
    (f) Three thousands
    (g) Eight units
    (h) No hundreds

12. (a) 1011
    (b) 1100
    (c) 1101
    (d) 1110

13. (a) 5
    (b) 6
    (c) 10
    (d) 9
    (e) 14
    (f) 27
    (g) 25
    (h) 33
    (i) 37
    (j) 52
    (k) 63
    (l) 64

14. (a) 101
    (b) 110
    (c) 1001
    (d) 1010
    (e) 10100
    (f) 11110
    (g) 110001
    (h) 1000110
    (i) 1100100

15.

	Base two	Base ten
(a)	1010	10
(b)	10011	19
(c)	11011	27
(d)	101110	46

16.

(a)	○	○	○	●	○	●
(b)	○	○	●	○	○	●
(c)	○	○	●	●	●	○
(d)	○	●	○	●	○	●
(e)	●	○	●	●	●	●
(f)	●	●	●	●	○	○

17.

(a)	○	○	○	●	●	○
(b)	○	○	●	○	●	●
(c)	○	●	○	○	○	●
(d)	○	●	●	●	○	●
(e)	●	○	●	○	○	●
(f)	●	●	●	○	○	●

5. TOWARDS A NUMBER LINE

1. (a) $3 < 7$; $7 > 3$
   (b) $9 > 5$; $5 < 9$
   (c) $1 < 6$; $6 > 1$
   (d) $6 < 8$; $8 > 6$
   (e) $5 < 7$; $7 > 5$

2. (a) $7 > 2$; $2 < 7$
   (b) $3 > 1$; $1 < 3$
   (c) $4 > 0$; $0 < 4$
   (d) $5 < 8$; $8 > 5$
   (e) $4 > 3$; $3 < 4$
   (f) $3 < 5$; $5 > 3$
   (g) $0 < 7$; $7 > 0$
   (h) $2 < 5$; $5 > 2$
   (i) $99 < 100$; $100 > 99$

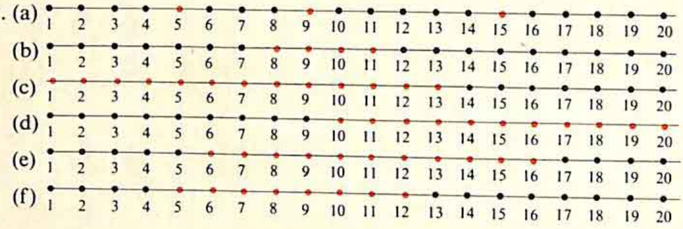
ANSWERS

3. (a) $2 < 4 < 7$
   (b) $1 < 5 < 6$
   (c) $3 < 7 < 10$
   (d) $5 < 6 < 9$
   (e) $9 < 10 < 11$
   (f) $4 < 5 < 6$

4. (a) $1 < 2 < 3 < 4$
   (b) $7 < 8 < 9 < 10$
   (c) $6 < 7 < 8 < 9 < 10$
   (d) $5 < 6 < 7 < 8 < 9$
   (e) $1 < 2 < 3 < 4 < 5$
   (f) $1 < 2 < 3 < 4 < 5 < 6 < 7 < 8 < 9 < 10$

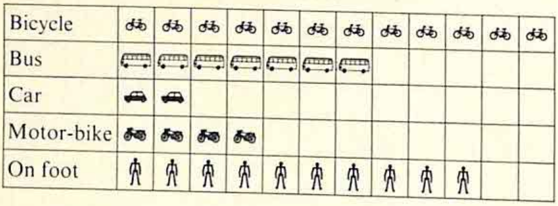
5. (a) Cardinal: 2; 1400; 100; 3000
   (b) Ordinal: 27; 1883

7.



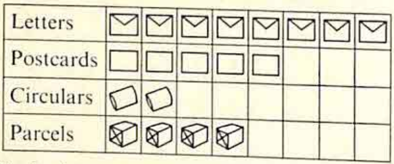
6. USING A NUMBER LINE

3. A graph to show how 35 people get to their office



4. (a) 70; (b) 50; (c) 20; (d) 30

5. A graph to show a firm's morning mail

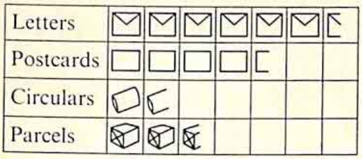


Each picture represents 10 items

S.C.E.R.T., West Bengal
Library
Calcutta
25/3, B. C. Road

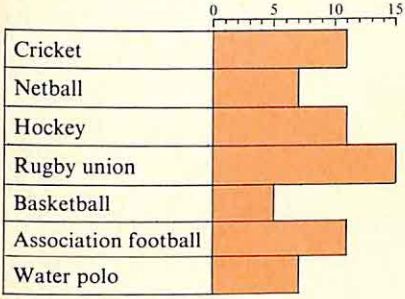
ANSWERS

6. A graph to show the mail on another day

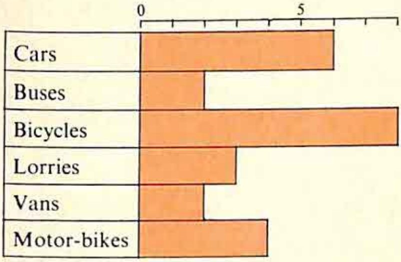


Each complete picture represents 10 items

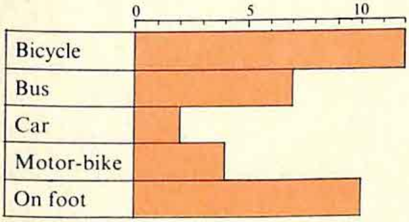
9. (a) A graph to show the number of players in a team



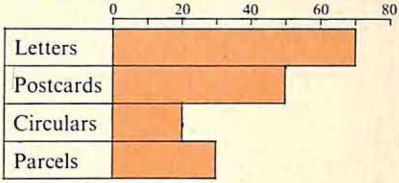
(b) A graph to show the traffic along a road in a period of ten minutes



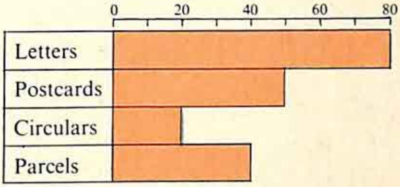
(c) A graph to show how 35 people get to their office



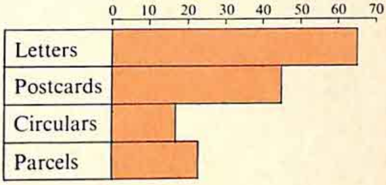
(d) A graph to show a firm's morning mail



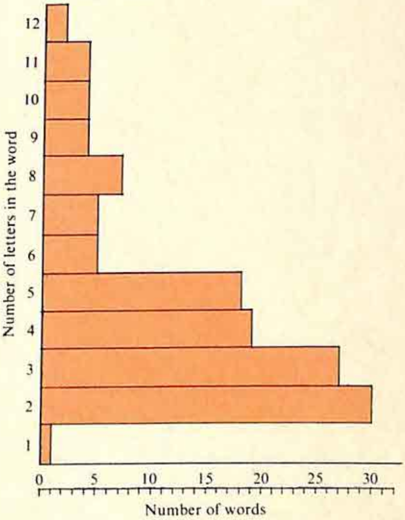
(e) A graph to show another firm's morning mail



(f) A graph to show the mail on another day



12. A graph to show the number of letters in the words of two paragraphs



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